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AEE 431 - Propulsion Systems II - Assignment I

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ABSTRACT

In this assignment paper, firstly the trend of the "Ratio of specific heats γ vs temperature for JP-8 and air combustion products" graph is interpreted and explained. Secondly compressor efficiency and compressor temperature ratio is derived in terms of pressure ratio and polytropic efficiency.

INTRODUCTION

The graph given below represents relation between ratio of specific heats γ and temperature and also contains data of the same relation for different fuel/air ratios for fuel JP-8 . The gamma values decreasing with the increasing temperature for both fuel contained and non-fuel cases. The gamma values also decreases with the increasing fuel/air ratio.

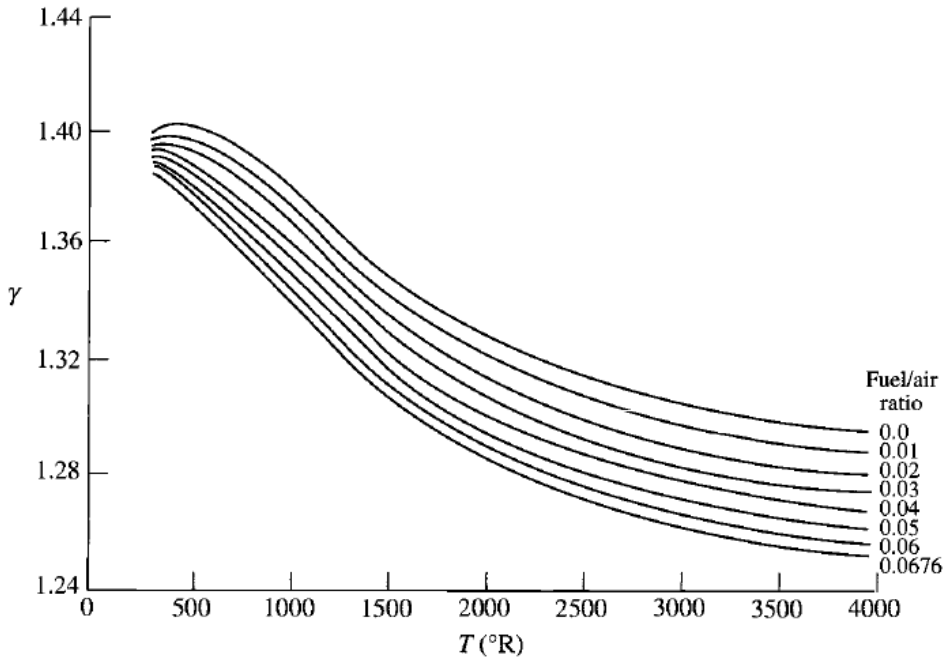

Figure 0.1: Ratio of specific heats γ, vs temperature for JP-8 and air combustion products.[1]

1 Part-1

Why increasing temperature leads to decreasing gamma?

The specific heat ratio (γ) is defined as the ratio of the specific heat at constant pressure (c_p) to the specific heat at constant volume (c_v):

$$\gamma = \frac{c_p}{c_v} \tag{1}$$

To understand the reduction of gamma, variation of c_p and c_v with temperature should be inspected. Both of the specific heats are functions of the temperature and with the increasing temperature both of them also increase for ideal gases. For ideal gas assumption there are accurate approaches for specific heats, based on experimental data or calculations from statistical attitude of gas molecules. For example third order polynomial equation from Çengel's Thermodynamics [**?**] book could be used for investigation.

TABLE A–2

Ideal–gas specific heats of various common gases (*Concluded*)

(*c*) As a function of temperature

$\bar{c}_p = a + bT + cT^2 + dT^3$
(T in K, c_p in kJ/kmol·K)

Substance	Formula	a	b	c	d	Temperature range, K	% error Max.	% error Avg.
Nitrogen	N_2	28.90	-0.1571×10^{-2}	0.8081×10^{-5}	-2.873×10^{-9}	273–1800	0.59	0.34
Oxygen	O_2	25.48	1.520×10^{-2}	-0.7155×10^{-5}	1.312×10^{-9}	273–1800	1.19	0.28
Air	—	28.11	0.1967×10^{-2}	0.4802×10^{-5}	-1.966×10^{-9}	273–1800	0.72	0.33
Hydrogen	H_2	29.11	-0.1916×10^{-2}	0.4003×10^{-5}	-0.8704×10^{-9}	273–1800	1.01	0.26
Carbon								

Figure 1.1: Table A–2 from Çengel's Thermodynamics book [**?**]

With the given polynomial, molar specific heat for constant pressure of air could be estimated in given temperature range as:

$$\bar{c_p} = 28.11 + 0.1967 \cdot 10^{-2} T + 0.4802 \cdot 10^{-5} T^2 - 0.1966 \cdot 10^{-9} T^3 \quad (kJ/(kmol \cdot K)) \tag{2}$$

To obtain specific heat for constant pressure of air, molar specific heat could divided by molar mass of air as:

$$c_p = \frac{\bar{c_p}}{M} \tag{3}$$

Molar mass of air is given in Çengel's book as 28.97 kg/kmol and gas constant of air could be find as 0.287 kJ/kg·K from universal gas constant divided by molar mass of air.

To find c_v values, ideal gas relation could be used:

$$c_v = c_p - R \tag{4}$$

With equations 1,2,3 and 4, the temperature-dependent change of c_p, c_v and γ can be plotted for given temperature range as follows:

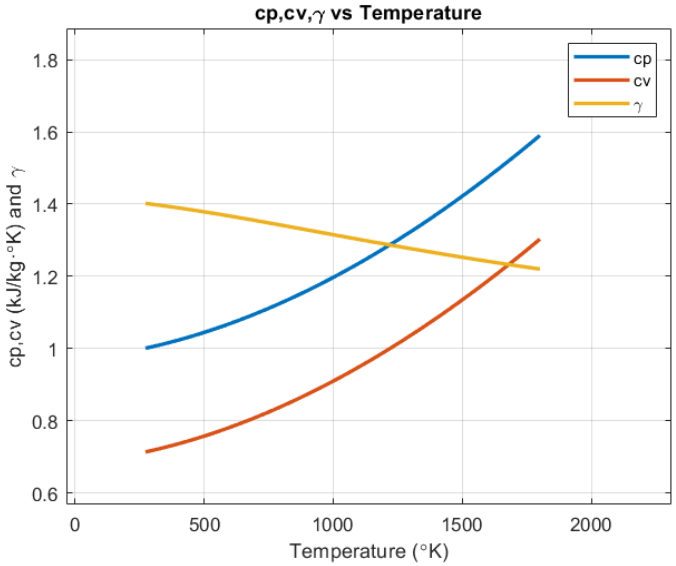

Figure 1.2: c_p, c_v, γ vs Temperature

Simply, if the results are examined, it will be seen that although the specific heats increase, the difference between them does not change with respect to the ideal gas assumption. Mathematically if the numerator is greater than the denominator for fractions with the same numerator and denominator difference then fraction which have greater denominator is less than other.

Comment:The specific heats c_p and c_v are both temperature-dependent, and their values vary with temperature because of the energy storage mechanisms in molecules. To understand why γ decreases with increasing temperature, we must explore the molecular basis for the specific heats. Reference 3 contains detailed information about molecular aspect of heat transactions and explains this phenomenon with additional degrees of freedom (rotational and vibrational)in the gas molecules [**?**]. However, this paper considers gases as ideal gases and does not inspect them as much detail as molecular physics.

2 PART-2

WHY INCREASING FUEL RATIO LEADS TO DECREASING GAMMA IN THE SAME TEMPERATURE?

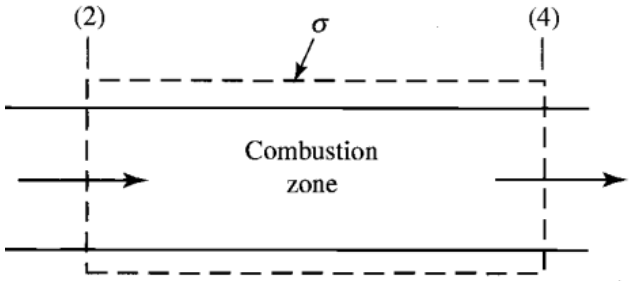

Figure 2.1: Combustor Model [1]

Application of the steady flow energy equation (first law of thermodynamics) to the control volume about the burner shown in Figure 1 gives:

$$\dot{m}_0 h_{t2} + \dot{m} f h_{PR} = (\dot{m}_0 + \dot{m}_f) h_{t4} \tag{5}$$

where h_{PR} is the thermal energy released by the fuel during combustion. For an ideal engine,

$$\dot{m}_0 + \dot{m}_f \cong \dot{m}_0 \tag{6}$$

Thus the preceding equation becomes

$$\dot{m}_0 c_{pc} T_{t2} + \dot{m}_f h_{PR} = \dot{m}_0 c_{pt} T_{t4} \tag{7}$$

c_{pc}=compressor exit c_p
c_{pt}=turbine exit c_p

$$\dot{m}_f h_{PR} = \dot{m}_0 (T_{t4} c_{pt} - T_{t2} c_{pc}) \tag{8}$$

The fuel/air ratio f is defined as

$$f \equiv \frac{\dot{m}_f}{\dot{m}_0} = \frac{c_{pc} T_{t2}}{h_{PR}} \left(\frac{T_{t4} c_{pt}}{T_{t2} c_{pc}} - 1 \right) \tag{9}$$

c_{pt}/c_{pc} ratio from Eq(6.2) in reference [1];

$$\frac{c_{pt}}{c_{pc}} = \frac{\gamma_t}{\gamma_t - 1}\frac{\gamma_c - 1}{\gamma_c} \tag{6.2}$$

Substitute equation 6.2 into equation 9;

$$f \equiv \frac{\dot{m}_f}{\dot{m}_0} = \frac{c_{pc}T_{t2}}{h_{PR}}\left(\frac{T_{t4}}{T_{t2}}\left(\frac{\gamma_t}{\gamma_t - 1}\frac{\gamma_c - 1}{\gamma_c}\right) - 1\right) \tag{10}$$

c_{pc} and T_{t2} is constant.

If the numerator is greater than the denominator by one, we obtain larger results from the ratios of smaller numbers compared to the ratios of larger numbers.Therefore, when we want to obtain a constant temperature when the fuel ratio increases (T_{t4} is constant), we expect γ_t to decreas.(γ_c is constant).

3 PART-3

DERIVATION COMPRESSOR EFFICIENCY AND COMPRESSOR TEMPERATURE RATIO.

From Mattingly and Ohain's book[1], compressor polytropic efficiency, e_c, is defined as:

$$e_c = \frac{ideal\ work\ of\ compression\ for\ a\ differential\ pressure\ change}{actual\ work\ of\ compression\ for\ a\ differential\ pressure\ change}$$

$$e_c = \frac{dw_i}{dw} = \frac{dh_{ti}}{dh_t} = \frac{dT_{ti}}{dT_t}$$

PV^γ = constant → Isentropic process
$PV = RT$ → Ideal gas law (per kg of gas)

$$\frac{P_3}{P_2} = \left(\frac{T_3}{T_2}\right)^{\frac{\gamma}{\gamma-1}} \rightarrow \frac{T_2^\gamma}{P_2^{\gamma-1}} = \frac{T_3^\gamma}{P_3^{\gamma-1}} \rightarrow T_2^\gamma P_2^{\gamma-1} = T_3^\gamma P_3^{\gamma-1} \rightarrow T_{ti}^\gamma P_{ti}^{1-\gamma} = \text{constant}$$

Infinitesimal temperature change

$$T^\gamma P^{1-\gamma} = \text{constant}$$

Exponentiate with base e:

$$\log_e (T^\gamma P^{1-\gamma}) = \log_e constant \rightarrow \log_e T^\gamma + \log_e P^{1-\gamma} = \log_e constant$$

$$\gamma \ln T + (1-\gamma) \ln P = \ln (constant)$$

Now, let's differentiate,

$$\gamma \frac{dT_{ti}}{T_t} + (1-\gamma)\frac{dP_t}{P_t} = 0 \rightarrow \frac{dT_{ti}}{T_t} = \frac{\gamma-1}{\gamma}\frac{dP_t}{P_t} \rightarrow \text{Isentropic Process}$$

Let's introduce a similar concept for polytropic efficiency:

$$PV^\gamma = constant \rightarrow \text{Isentropic Process}$$

$$PV^n = constant \rightarrow \text{Polytropic Process} \tag{11}$$

The relationship given by Eq(11) describes a polytropic process that is expansion and compression processes which include heat transfer, where p is pressure, V is volume and n is poytropic index. For an isenthalpic process, the value of n is equal to 0. For a isothermal process, the value of n is equal to 1

$$T^n P^{1-n} = constant$$

Then,

$$\frac{dT_t}{T_t} = \frac{n-1}{n}\frac{dP_t}{P_t} \rightarrow \text{Polytropic Process}$$

$$e_c = \frac{dT_{ti}}{dT_t} = \frac{\frac{\gamma-1}{\gamma}\frac{dP_t}{P_t}T_t}{\frac{n-1}{n}\frac{dP_t}{P_t}T_t} = \frac{\frac{\gamma-1}{\gamma}}{\frac{n-1}{n}} = \frac{\gamma-1}{\gamma}\frac{n}{n-1}$$

Recalling compressor efficiency:

$$\eta_c = \frac{\pi^{\frac{\gamma-1}{\gamma}} - 1}{\tau_c - 1}$$

Given that:

$$e_c = \frac{\gamma-1}{\gamma}\frac{n}{n-1} \quad \xrightarrow{\text{a simple touch}} \quad \frac{n-1}{n} = \frac{\gamma-1}{\gamma}\frac{1}{e_c}$$

Thus,

$$\tau_c = \pi_c^{\frac{n-1}{n}} = \pi_c^{\frac{\gamma-1}{\gamma}\frac{1}{e_c}}$$

Then, rewriting compressor efficiency:

$$\eta_c = \frac{\pi_c^{\frac{\gamma-1}{\gamma}} - 1}{\pi_c^{\frac{\gamma-1}{\gamma}\frac{1}{e_c}} - 1}$$

REFERENCES

[1] Mattingly, J., D., 2016, *Elements of Propulsion: Gas Turbines and Rockets,* 2nd ed., American Institute of Aeronautics and Astronautic, Reston, Virginia

[*] Oğuzhan Çamlıca, "splendiferous one ponders Formidable, hawk doth not pursue a gnat."