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AEE 431 - Propulsion Systems II - Assignment II

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ABSTRACT

In this assignment paper, we present two equations: the first one describes the static temperature ratio from turbojet exit to inlet, and the second one represents the turbine temperature ratio in a turbofan engine. We will demonstrate these equations using real cycle assumptions and known parametric cycle equations.

(Q1) Derive the expression given below[1] using parametric cycle analysis (PCA) of real engines, starting from the assumptions of Turbojet a real cycle analysis.

$$\frac{T_9}{T_0} = \frac{\tau_\lambda \tau_t}{(P_{t9}/P_9)^{((\gamma-1)/\gamma)}} \frac{c_{pc}}{c_{pt}} \tag{1}$$

Real Cycle Analysis Assumptions:

1. Perfect gas upstream of the main burner with constant properties, γ_c, R_c, c_{pc}, etc.
2. Perfect gas downstream of the main burner with constant properties, γ_t, R_t, c_{pt}, etc.
3. All components are adiabatic.
4. Polytropic efficiencies will be used for compressors, turbines, and fans.
5. No cooling or bleed flows.

Simple modifications with respect to isentropic relation:

$$\frac{T_9}{T_0} = \frac{T_{t9}/T_0}{T_{t9}/T_9} \longrightarrow \frac{T_{t9}/T_0}{(P_{t9}/P_9)^{((\gamma-1)/\gamma)}}$$

T_{t9}/T_0 could expressed as:

$$\frac{T_{t9}}{T_0} = \tau_r \tau_d \tau_c \tau_b \tau_t \tau_n$$

Nozzle is adiabatic:

$$\tau_n = 1$$

Since τ_λ is defined as the ratio of the burner exit enthalpy $c_p T_t$ to the ambient enthalpy $c_p T_0$:

$$\tau_\lambda = \frac{c_{pt}}{c_{pc}} \frac{T_{t4}}{T_0}$$

And T_{t4}/T_0 could given as:

$$\frac{T_{t4}}{T_0} = \tau_r \tau_d \tau_c \tau_b$$

So,

$$\frac{T_{t9}}{T_0} = \frac{T_{t4}}{T_0}\tau_t = \frac{c_{pc}}{c_{pt}}\tau_\lambda \tau_t \quad \xrightarrow{Finally} \quad \frac{T_9}{T_0} = \frac{\tau_\lambda \tau_t}{(P_{t9}/P_9)^{((\gamma-1)/\gamma)}} \frac{c_{pc}}{c_{pt}}$$

(Q2) Derive the expression given below[1] using parametric cycle analysis (PCA) of real engines, starting from the assumptions of a real turbofan cycle analysis.

$$\tau_t = 1 - \frac{1}{\eta_m(1+f)} \frac{\tau_r}{\tau_\lambda} [(\tau_c - 1) + \alpha(\tau_f - 1)] \tag{2}$$

Real Cycle Analysis Assumptions:

1. Perfect gas upstream of the main burner with constant properties, γ_c, R_c, c_{pc}, etc.
2. Perfect gas downstream of the main burner with constant properties, γ_t, R_t, c_{pt}, etc.
3. All components are adiabatic.
4. Polytropic efficiencies will be used for compressors, turbines, and fans.
5. No cooling or bleed flows.

The power balance between the turbine, compressor, and fan, with a mechanical efficiency η_m of the coupling between the turbine and compressor and fan, gives

$$\underbrace{\dot{m}_C c_{pc}(T_{t3} - T_{t2})}_{\text{Power into compressor}} + \underbrace{\dot{m}_F c_{pc}(T_{t13} - T_{t2})}_{\text{Power into fan}} = \underbrace{\eta_m \dot{m}_4 C_{pt}(T_{t4} - T_{t5})}_{\text{Net power from turbine}}$$

Taking parentheses of T_{t2}:

$$\dot{m}_C c_{pc} T_{t2}\left(\frac{T_{t3}}{T_{t2}} - 1\right) + \dot{m}_F c_{pc} T_{t2}\left(\frac{T_{t13}}{T_{t2}} - 1\right) = \quad \eta_m \dot{m}_4 C_{pt} T_{t2}\left(\frac{T_{t4}}{T_{t2}} - \frac{T_{t5}}{T_{t2}}\right)$$

Bypass ratio,

$$\alpha = \frac{\dot{m}_F}{\dot{m}_C}$$

And,

$$\dot{m}_0 = \dot{m}_C + \dot{m}_F = (1+\alpha)\dot{m}_C$$

Fuel/Air ratio,

$$f = \frac{\dot{m}_f}{\dot{m}_C}$$

Mass flow rate enters burner:

$$\dot{m}_4 = \dot{m}_C + \dot{m}_f = (1+f)\dot{m}_C$$

Dividing the preceding equation by $\dot{m}_C c_{pc} T_{t2}$,

$$\left(\frac{T_{t3}}{T_{t2}} - 1\right) + \alpha\left(\frac{T_{t13}}{T_{t2}} - 1\right) = \frac{c_{pt}}{c_{pc}}\eta_m(1+f)\left(\frac{T_{t4}}{T_{t2}} - \frac{T_{t5}}{T_{t2}}\right)$$

Compressor temperature ratio and fan temperature ratio:

$$\frac{T_{t3}}{T_{t2}} = \tau_c \quad \text{and} \quad \frac{T_{t13}}{T_{t2}} = \tau_f$$

Right hand side Temperature ratio terms could expressed as:

$$\frac{T_{t4}}{T_{t2}} = \frac{T_{t4}/T_0}{T_{t2}/T_0} = \frac{\frac{c_{pc}}{c_{pt}}\tau_\lambda}{\tau_r}$$

And,

$$\frac{T_{t5}}{T_{t2}} = \frac{T_{t5}/T_0}{T_{t2}/T_0} = \frac{(T_{t4}/T_0)(T_{t5}/(T_{t4}))}{T_{t2}/T_0} = \frac{\frac{c_{pc}}{c_{pt}}\tau_\lambda\tau_t}{\tau_r}$$

So preceding equation with temperature ratios becomes:

$$(\tau_c - 1) + \alpha(\tau_f - 1) = \quad \frac{c_{pt}}{c_{pc}}\eta_m(1+f)\left(\frac{\frac{c_{pc}}{c_{pt}}\tau_\lambda}{\tau_r} - \frac{\frac{c_{pc}}{c_{pt}}\tau_\lambda\tau_t}{\tau_r}\right) = \quad \eta_m(1+f)\frac{\tau_\lambda}{\tau_r}(1-\tau_t)$$

And finally for turbine temperature ratio:

$$\tau_t = 1 - \frac{1}{\eta_m(1+f)}\frac{\tau_r}{\tau_\lambda}\left[(\tau_c - 1) + \alpha(\tau_f - 1)\right]$$

REFERENCES

[1] Mattingly, J., D., 2016, *Elements of Propulsion: Gas Turbines and Rockets,* 2nd ed.,American Institute of Aeronautics and Astronautic, Reston, Virginia

[*] Albert Camus "The struggle itself toward the heights is enough to fill a man's heart. One must imagine Sisyphus happy."