ANKARA YILDIRIM BEYAZIT UNIVERSITY FACULTY OF AERONAUTICS AND ASTRONAUTICS DEPARTMENT OF AEROSPACE ENGINEERING

AEE 431 - Propulsion Systems II - Assignment III

Batuhan Şahin-20110151003
Muhammet Berati Kılıç-20110111028
Oğuzhan Çamlıca-20110111021
Arzu İsmi-20110111016

(Q1) Explanation of the statements found on page 10 of the week 4 slides

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- For constant fan pressure ratio of 1.6, fuel consumption decreases with increasing bypass ratio.
- For constant bypass ratio of 9, fuel consumption decreases with increasing fan pressure ratio.

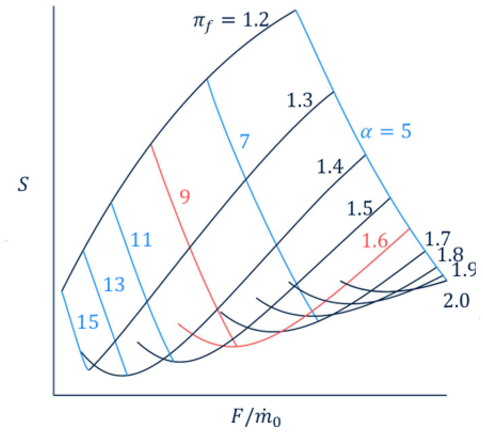

Figure 1: Specific Thrust to fuel consumption

Constant fan pressure where π_{fan}=1.6, and increasing bypass ratio $\alpha = \frac{\dot{m}_F}{\dot{m}_C}$ results in a decrease in fuel consumption S. Some equations from Matingly's[1] book are given as:

$$S = \frac{\dot{m}_f}{F_{\text{total}}} = \frac{\dot{m}_f}{F_C + F_F} \tag{1.1}$$

$$F_F = \dot{m}_F \frac{a_0}{g_c}\left(\frac{V_{19}}{a_0} - M_0 + \frac{T_{19}}{T_0}\frac{V_{19}}{a_0}\frac{1-\frac{P_0}{P_{19}}}{\gamma_c}\right) \tag{1.2}$$

$$F_C = \dot{m}_C \frac{a_0}{g_c}\left((1+f)\frac{V_9}{a_0} - M_0 + (1+f)\frac{R_t}{R_c}\frac{T_9}{T_0}\frac{V_9}{a_0}\frac{1-\frac{P_0}{P_9}}{\gamma_c}\right) \tag{1.3}$$

Using equation (1.2) and (1.3), increasing α increases thrust. However, from equation (1.1), specific thrust is decreasing as α increases.

For a constant bypass ratio of $\alpha = 9$, increasing the fan pressure ratio π_{f} decreases the fuel consumption S.

$$\tau_f = (\pi_f)^{\frac{\gamma_c}{\gamma_c - 1}} \tag{1.4}$$

Increasing the fan pressure ratio π_{fan} leads to an increase in the temperature ratio, which in turn increases the exit velocity of the fan. Referring to equation (1.2), this implies that the thrust generated by the fan increases. Consequently, using equation (1.1), the fuel consumption S decreases.

(Q2) Derivation of the equation on page 23 of week 4 slide.

$$f_{AB} = \left(1 + \frac{f}{1+\alpha}\right) \cdot \frac{h_{t7-h_{t6A}}}{\eta_{AB} h_{PR} - h_{t7}} \tag{2}$$

Applying energly balance to afterburner(between 6A and 7)

$$(\dot{m}_C + \dot{m}_B + \dot{m}_f) c_p T_{t6A} + \eta AB \dot{m_{f_{AB}}} h_{PR} = (\dot{m}_C + \dot{m}_B + \dot{m}_f + \dot{m_{f_{AB}}}) c_p T_{t7}$$

Then,

$$(\dot{m}_C + \dot{m}_B + \dot{m}_f) c_p T_{t6A} + \eta AB \dot{m_{f_{AB}}} h_{PR} = (\dot{m}_C + \dot{m}_B + \dot{m}_f +) c_p T_{t7} + \dot{m_{f_{AB}}} c_p T_{t7}$$

Dividing everywhere by $\dot{m}_O$ equation becomes,

$$\frac{\eta_{AB} \dot{m_{f_{AB}}} h_{PR}}{\dot{m}_O} - \frac{\dot{m_{f_{AB}}} c_p T_{t7}}{\dot{m}_O} = \frac{(\dot{m}_C + \dot{m}_B + \dot{m}_f) c_p T_{t7}}{\dot{m}_O} - \frac{(\dot{m}_C + \dot{m}_B + \dot{m}_f) c_p T_{t6A}}{\dot{m}_O}$$

$$\frac{\dot{m_{f_{AB}}}}{\dot{m}_O} (\eta_{AB} h_{PR} - c_p T_{t7}) = \frac{(\dot{m}_C + \dot{m}_B + \dot{m}_f) c}{\dot{m}_O} c_p (T_{t7} - T_{t6A})$$

Afterburner fuel/air ratio:

$$f_{AB} = \frac{\dot{m_{f_{AB}}}}{\dot{m}_O}$$

Fuel/air ratio:

$$f = \frac{\dot{m}_f}{\dot{m}_O} \quad \text{And,} \quad \dot{m}_O = \dot{m}_C + \dot{m}_B$$

Equation becomes,

$$f_{AB} (\eta_{AB} h_{PR} - c_p T_{t7}) = \frac{(\dot{m}_C + \dot{m}_B + \dot{m}_f)}{\dot{m}_C + \dot{m}_B} c_p (T_{t7} - T_{t6A})$$

Manupulating the right part of the equation,

$$\frac{(\dot{m}_C + \dot{m}_B + \dot{m}_f)}{\dot{m}_C + \dot{m}_B} = \frac{(\dot{m}_C + \dot{m}_B)}{\dot{m}_C + \dot{m}_B} + \frac{\dot{m}_f}{\dot{m}_C + \dot{m}_B} = 1 + \frac{\frac{\dot{m}_f}{\dot{m}_C}}{\frac{(\dot{m}_C + \dot{m}_B)}{\dot{m}_C}} = 1 + \frac{f}{1+\alpha}$$

Then,

$$f_{AB} (\eta_{AB} h_{PR} - c_p T_{t7}) = \left(1 + \frac{f}{1+\alpha}\right) c_p (T_{t7} - T_{t6A})$$

And,

$$f_{AB} = \left(1 + \frac{f}{1+\alpha}\right) \frac{c_p (T_{t7} - T_{t6A})}{\eta_{AB} h_{PR} - c_p T_{t7}}$$

Finally afterburner fuel/air ratio:

$$f_{AB} = \left(1 + \frac{f}{1+\alpha}\right) \frac{h_{t7} - h_{t6A}}{\eta_{AB} h_{PR} - h_{t7}}$$

(Q3) Derivation of the equation on page 24 of week 4 slide.

$$\tau_t = 1 - \frac{1}{\eta_m(1+f)}\frac{\tau_r}{\tau_\lambda}[(\tau_c - 1) + \alpha(\tau_f - 1)] \tag{3}$$

Real Cycle Analysis Assumptions:

1. Perfect gas upstream of the main burner with constant properties, γ_c, R_c, c_{pc}, etc.
2. Perfect gas downstream of the main burner with constant properties, γ_t, R_t, c_{pt}, etc.
3. All components are adiabatic.
4. Polytropic efficiencies will be used for compressors, turbines, and fans.
5. No cooling or bleed flows.

The power balance between the turbine, compressor, and fan, with a mechanical efficiency η_m of the coupling between the turbine and compressor and fan, gives

$$\underbrace{\dot{m}Ccpc(T_{t3} - T_{t2})}_{\text{Power into compressor}} + \underbrace{\dot{m}Fcpc(T_{t13} - T_{t2})}_{\text{Power into fan}} = \underbrace{\eta_m \dot{m}4Cpt(T_{t4} - T_{t5})}_{\text{Net power from turbine}}$$

Taking parentheses of T_{t2}:

$$\dot{m}CcpcT_{t2}\left(\frac{T_{t3}}{T_{t2}} - 1\right) + \dot{m}FcpcT_{t2}\left(\frac{T_{t13}}{T_{t2}} - 1\right) = \quad \eta_m \dot{m}4CptT_{t2}\left(\frac{T_{t4}}{T_{t2}} - \frac{T_{t5}}{T_{t2}}\right)$$

Bypass ratio,

$$\alpha = \frac{\dot{m}_F}{\dot{m}_C}$$

And,

$$\dot{m}_0 = \dot{m}_C + \dot{m}_F = (1+\alpha)\dot{m}_C$$

Fuel/Air ratio,

$$f = \frac{\dot{m}_f}{\dot{m}_C}$$

Mass flow rate enters burner:

$$\dot{m}_4 = \dot{m}_C + \dot{m}_f = (1+f)\dot{m}_C$$

Dividing the preceding equation by $\dot{m}CcpcT_{t2}$,

$$\left(\frac{T_{t3}}{T_{t2}} - 1\right) + \alpha\left(\frac{T_{t13}}{T_{t2}} - 1\right) = \frac{c_{pt}}{c_{pc}}\eta_m(1+f)\left(\frac{T_{t4}}{T_{t2}} - \frac{T_{t5}}{T_{t2}}\right)$$

Compressor temperature ratio and fan temperature ratio:

$$\frac{T_{t3}}{T_{t2}} = \tau_c \quad \text{and} \quad \frac{T_{t13}}{T_{t2}} = \tau_f$$

Right hand side Temperature ratio terms could expressed as:

$$\frac{T_{t4}}{T_{t2}} = \frac{T_{t4}/T_0}{T_{t2}/T_0} = \frac{\frac{c_{pc}}{c_{pt}}\tau_\lambda}{\tau_r}$$

And,

$$\frac{T_{t5}}{T_{t2}} = \frac{T_{t5}/T_0}{T_{t2}/T_0} = \frac{(T_{t4}/T_0)(T_{t5}/(T_{t4}))}{T_{t2}/T_0} = \frac{\frac{c_{pc}}{c_{pt}}\tau_\lambda\tau_t}{\tau_r}$$

So preceding equation with temperature ratios becomes:

$$(\tau_c - 1) + \alpha(\tau_f - 1) = \frac{c_{pt}}{c_{pc}}\eta_m(1+f)\left(\frac{\frac{c_{pc}}{c_{pt}}\tau_\lambda}{\tau_r} - \frac{\frac{c_{pc}}{c_{pt}}\tau_\lambda\tau_t}{\tau_r}\right) = \eta_m(1+f)\frac{\tau_\lambda}{\tau_r}(1-\tau_t)$$

And finally for turbine temperature ratio:

$$\tau_t = 1 - \frac{1}{\eta_m(1+f)}\frac{\tau_r}{\tau_\lambda}\left[(\tau_c - 1) + \alpha(\tau_f - 1)\right]$$

(Q4) Explain the trend of the graph of specific thrust and thrust specific fuel consumption versus compressor pressure ratio given below.

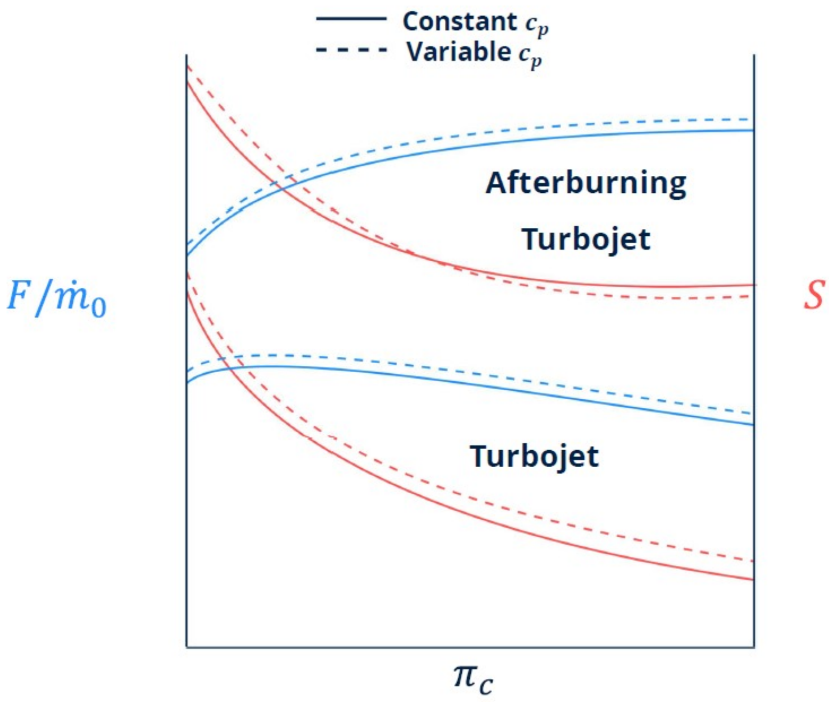

Figure 2: Compressor pressure ratio vs specific thrust and thrust specific fuel consumption

As the compressor pressure ratio, π_c of a turbojet increases, the total pressure and total temperature of the air entering the combustion chamber also rise. This requires a lower fuel-to-air ratio to achieve the same burner exit total temperature, which in turn reduces the thrust-specific fuel consumption. However, since the compressor pressure ratio increases, the work required to drive the compressor also rises. As a result, the turbine work output must increase, reducing the difference between the turbine exit total temperature and the nozzle total temperature. This decrease in temperature difference reduces the exit velocity, which leads to a reduction in both thrust and specific thrust. For an afterburner turbojet, the effect on thrust specific fuel consumption remains the same, with the only difference being the additional fuel required for the afterburner. In such engines, afterburner combustion increases thrust, and consequently, specific thrust also increases.

(Q5) For a single-spool turbojet engine with losses, determine the compressor exit temperature and pressure, the turbine exit temperature and pressure, and the nozzle exit Mach number for the input in Table 1.

Table 0.1: Turbojet input data

$M_0 = 1.2$	$\pi_c = 9.2$	$T_{t4} = 1810\,\mathrm{K}$
$P_0 = 28.53\,\mathrm{kPa}$	$T_0 = 233\,\mathrm{K}$	$h_{PR} = 42800\,\mathrm{kJ/kg}$
$c_{pc} = 1.002\,\mathrm{kJ/(kgK)}$	$c_{pt} = 1.357\,\mathrm{kJ/(kgK)}$	$\gamma_c = 1.3$
$\pi_b = 0.915$	$\eta_b = 0.99$	$\gamma_t = 1.3$
$P_0/P_9 = 0.85$	$\pi_{dmax} = 0.95$	$e_c = 0.84$
		$e_t = 0.91$
		$\eta_m = 0.98$
		$\pi_n = 0.98$

From the 3rd iteration, for 1810 K and f = 0.03669:

$$h_{t4} = 2124.5671, \quad f_{new} = 0.036704, \quad \Delta f = 1.4132 * 10^{-2}$$

Iteration 4: For 1810 K and f=0.036704, from the Table D, h_{t4} is:

$$\frac{0.0507 - 0.0338}{0.036704 - 0.0338} = \frac{2164.45 - 2116.34}{h_{t4} - 2116.34}$$

$$h_{t4} = 2124.6$$

Then,

$$f_{new} = \frac{2124.6 - 647.32}{0.99 \cdot 42800 - 2124.6} = 0.036705$$

$$\Delta f = 9.797 \cdot 10^{-7} < 10^{-5}$$

This part is rest of the solution. Some values are already calculated. To find exit temperature of compressor,

$$T_{t3} = T_0 \tau_r \tau_d \tau_c$$

so,

$$T_{t3} = 552.13\,K$$

To find exit pressure of compressor,

$$P_{t3} = P_0 \pi_r \pi_d \pi_c$$

so,

$$P_{t3} = 740.269\,kPa$$

To find the turbine exit temperature of turbine,

$$T_{t5} = T_0 \tau_r \tau_d \tau_c \tau_b \tau_t$$

$$\tau_t = 1 - \frac{1}{\eta_m (1+f)} \frac{\tau_r}{\tau_\lambda} (\tau_c - 1) \tag{5.1}$$

$$\tau_\lambda = \frac{c_{pt} T_{t4}}{c_{pc} T_0} \tag{5.2}$$

So,

$$\tau_t = 0.743664864 \quad and \quad T_{t5} = 1346.033\,K$$

To find exit pressure of turbine,

$$P_{t5} = P_0 \pi_r \pi_d \pi_c \pi_b \pi_t$$

$$\pi_t = \tau_t^{\frac{\gamma_t}{(\gamma_t - 1) e_t}} \tag{5.3}$$

So,

$$\pi_t = 0.244068962 \quad and \quad P_{t5} = 165.319\,kPa$$

To find exit Mach number,

$$M_9 = \sqrt{\frac{2}{\gamma_t - 1} \left[(\frac{P_{t9}}{P_9})^{\frac{\gamma_t - 1}{\gamma_t}} - 1 \right]} \tag{5.4}$$

$$\frac{P_{t9}}{P_9} = \frac{P_0}{P_9} \pi_r \pi_d \pi_c \pi_b \pi_t \pi_n \tag{5.5}$$

So,

$$\frac{P_{t9}}{P_9} = 4.82686113 \quad and \quad M_9 = 1.70886$$

REFERENCES

[1] Mattingly, J., D., 2016, *Elements of Propulsion: Gas Turbines and Rockets,* 2nd ed.,American Institute of Aeronautics and Astronautic, Reston, Virginia

*Gorgias "Nothing exists; even if something exists, nothing can be known about it; and even if something can be known about it, knowledge about it can't be communicated to others. Even if it can be communicated, it cannot be understood"