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Maximizing Lift-to-Drag Ratio of 4-Digit NACA Airfoils Through Sequential Quadratic Programming

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Abstract

This study focuses on the aerodynamic optimization of NACA 4-digit airfoils under geometric and performance constraints. The optimization problem involves maximizing c_l/c_d by selecting optimal design parameters, including maximum camber, camber location, maximum thickness of the airfoil and angle of attack. The Sequential Quadratic Programming (SQP) method in MATLAB was employed for optimization, while airfoil data such as c_l, c_d, and c_m were obtained using XFOIL. The results demonstrate that the c_l/c_d ratio, a critical parameter in aerodynamics and aircraft design, was successfully maximized. This achievement highlights the potential for achieving high aerodynamic performance through systematic optimization techniques. Also, the approach offers a systematic methodology for improving airfoil performance, contributing valuable insights to aerodynamic design processes.

Keywords: Aerodynamic optimization; NACA 4-digit airfoils; cl/cd ratio; Sequential Quadratic Programming (SQP); XFOIL

Nomenclature

c_l Lift coefficient
c_d Drag coefficient
c_m Moment coefficient
c_l/c_d Lift-to-drag ratio
m Maximum camber
p Position of maximum camber
t Maximum thickness
α Angle of attack
Re Reynolds number
$Mach$ Mach number
θ Angle of inclination of the camber line
x Chordwise position along the airfoil
y Vertical position along the airfoil
y_t Thickness distribution
y_c Camber line
$\frac{dy}{dx}$ Gradient of the camber line
d Search direction in optimization
$f(x)$ Objective function
$L(x, \lambda, \mu)$ Lagrangian function
λ Lagrange multiplier for equality constraints
μ Lagrange multiplier for inequality constraints
Δx_k Perturbation in design variables

∇ Gradient operator
∇^2 Hessian matrix operator
XFOIL Panel method-based aerodynamic analysis tool
SQP Sequential Quadratic Programming
NACA National Advisory Committee for Aeronautics

1. Introduction

Aerodynamic optimization plays a crucial role in enhancing the performance and efficiency of modern aerodynamic systems. One of the primary objectives in airfoil optimization is maximizing the lift-to-drag ratio (c_l/c_d), which directly impacts fuel efficiency, range, and overall operational performance. Airfoil geometry has a significant influence on aerodynamic characteristics, making it a critical focus of research in both theoretical and applied aerodynamics.

Several optimization techniques have been developed to address this challenge. Among these, Genetic Algorithms (GA) and Sequential Quadratic Programming (SQP) are two widely used methods. Genetic Algorithms are stochastic, population-based methods inspired by natural selection, offering robust solutions to complex and multi-modal problems. In contrast, SQP is a deterministic, gradient-based optimization technique that is particularly effective for solving nonlinear programming problems with constraints.

Previous studies have demonstrated the effectiveness of both methods in aerodynamic optimization. For example, the work by [1] investigated the aerodynamic optimization of a UAV wing under multiple constraints, including weight, geometry, root bending moment, and performance parameters. This study highlighted the utility of optimization techniques in achieving high-performance designs. Similarly, [2] compared the performance of Genetic Algorithms and Sequential Quadratic Programming in optimizing NACA four-digit airfoils. The results showed that while GAs are better suited for exploring a broader design space, SQP offers faster convergence for problems with well-defined gradients.

This study focuses on the aerodynamic optimization of NACA 4-digit airfoils under geometric and performance constraints. The optimization process involves the use of XFOIL, a computational tool widely employed for analyzing airfoil aerodynamics, to calculate lift and drag coefficients. XFOIL integrates panel methods with boundary layer analysis to predict aerodynamic characteristics such as c_l, c_d, and c_m with high accuracy, making it a valuable tool in airfoil optimization studies. MATLAB serves as the main platform to automate the geometry generation, simulation execution, and iterative optimization process. A Sequential Quadratic Programming (SQP) method is utilized to adjust design parameters, specifically the maximum camber (m), camber position (p), and thickness (t), in order to maximize the airfoil's c_l/c_d ratio at a variable angle of attack. The optimum angle of attack is at the maximum c_l/c_d ratio.

By leveraging these tools and methodologies, this work seeks to further validate the effectiveness of systematic optimization techniques in airfoil design and provide valuable insights into their practical applications. The results of this study not only demonstrate the potential of optimization techniques in achieving high-performance airfoil geometries but also contribute to the broader field of aerodynamic optimization.

2. Method

2.1 Generation of 4-Digit NACA Airfoils

$$NACA \quad MPXX$$

This section describes the process of generating a 4-digit NACA airfoil using MATLAB. The method takes the user-provided parameters `M`, `P`, and `XX`, which define the maximum camber, the position of maximum camber, and the maximum thickness of the airfoil, respectively.[3]

2.1.1 Definition of Parameters

The NACA airfoil parameters are converted as follows:

- Maximum camber: $m = \frac{M}{100}$,
- Position of maximum camber: $p = \frac{P}{10}$,
- Maximum thickness: $t = \frac{XX}{100}$.

The airfoil's coordinates are generated using the following equations:

2.1.2 Thickness Distribution

The thickness distribution y_t is given by:

$$y_t = \frac{t}{0.2}\left(0.2969\sqrt{x} - 0.126x - 0.3516x^2 + 0.2843x^3 - 0.1015x^4\right), \tag{1}$$

where x represents the chordwise position along the airfoil.

2.1.3 Camber Line and Gradient

The camber line y_c and its gradient $\frac{dy}{dx}$ depend on the position of x relative to p:

$$y_c = \begin{cases} \frac{m}{p^2}\left(2px - x^2\right), & \text{if } x < p, \\ \frac{m}{(1-p)^2}\left((1-2p) + 2px - x^2\right), & \text{if } x \geq p, \end{cases} \tag{2}$$

$$\frac{dy}{dx} = \begin{cases} \frac{2m}{p^2}(p - x), & \text{if } x < p, \\ \frac{2m}{(1-p)^2}(p - x), & \text{if } x \geq p. \end{cases} \tag{3}$$

2.1.4 Upper and Lower Surface Coordinates

The upper and lower surfaces are calculated using:

$$x_{\text{upper}} = x - y_t \sin(\theta), \quad y_{\text{upper}} = y_c + y_t \cos(\theta), \tag{4}$$

$$x_{\text{lower}} = x + y_t \sin(\theta), \quad y_{\text{lower}} = y_c - y_t \cos(\theta), \tag{5}$$

where $\theta = \tan^{-1}\left(\frac{dy}{dx}\right)$.

2.1.5 Panel Distribution: Refining the Distribution Near Leading Edge and Trailing Edge

In computational aerodynamic analyses such as airfoil design the distribution of panels plays a crucial role. By refining the panel placement near the leading edge and trailing edge, we are be able to ensure that the aerodynamic characteristics are captured more accurately in these regions.

Cosine distribution method is one of the most commonly used methodsto increase the panel density in these critical regions. By using this method, the panels can be concentrated near the leading edge and trailing edge. The method is implemented as follows:

$$x_i = \frac{1}{2}\left(1 - \cos\left(\frac{i\pi}{n}\right)\right), \quad i = 1, 2, \ldots, n$$

Here, x_i represents the distribution of each panel along the chord length, while

n is the total number of panels. This formula ensures that the panels are placed more densely at both the leading and trailing edges. This method applied in MATLAB code by taking a number of panel from the user. The number of panel is also critical because, the accuracy of the result are depend on the number of panels.

XFOIL is a potential flow software that employs a panel method to model the airfoil, utilizing between 160 and 400 panels for computational purposes.[4]

After a number of panel the results cannot change and even it can cause a bad prediction of the flow. [5]

The cosine distribution method is commonly used because it focuses the panels around the leading and trailing edges, providing a more precise depiction of the flow characteristics in these areas. Unlike a uniform linear distribution that places panels uniformly across the chord and may miss important flow details in critical regions, the cosine distribution method allocates panels non-linearly, providing greater resolution in areas where the flow undergoes the most significant changes.[6]

The implementation of this method for increasing panel density in the critical regions offers several important advantages:

- Higher Resolution: By using cosine distribution, the resolution of the airflow representation increases, particularly near the leading edge and trailing edge, allowing for more accurate aerodynamic analysis.
- More Accurate Lift and Drag Calculations: Properly resolving the aerodynamic forces in these regions leads to more accurate calculations of the lift and drag coefficients.
- Capturing Flow Separation and Speed Changes: The enhanced panel density near the leading edge and trailing edge allows for better modeling of flow separations and rapid changes in flow speed.

This method is fundamental for understanding and optimizing airfoil aerodynamic performance. Therefore, methods like cosine distribution are widely used in advanced design processes to improve the accuracy of aerodynamic analyses.

2.1.6 Generated Airfoil

The airfoil coordinates are saved in a file for use in XFOIL. A sample plot of the airfoil shape is shown in Figure 1.

2.2 Calculation of c_l, c_d, c_m and Their Derivatives Using XFOIL

The lift coefficient (c_l), drag coefficient (c_d), and moment coefficient (c_m) are computed using the XFOIL software. The derivatives of these coefficients with respect to the design parameters (m, p, t, at a certain angle of attack, α) are then calculated using the central difference method.

Creating input file for XFOIL and calculation of aerodynamic coefficient

1. **Command File Creation:** An input file was created in matlab to calculate the aerodynamic coefficients in xfoil.
   A command file is written to automate the XFOIL execution. Key commands include:
```
PLOP G              % Disables graphical output to improve performance.
LOAD <filename>     % Loads the created and saved file specified file.
OPER                % Enters operational mode for aerodynamic analysis.
ITER <number>       % Sets the maximum number of iterations for convergence.
RE <Reynolds number>    % Defines the Reynolds number.
```

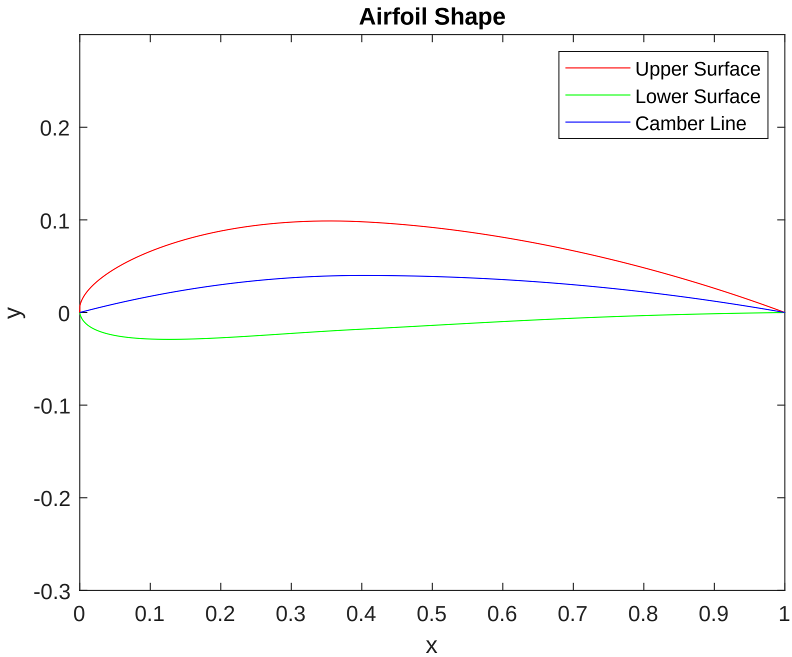

Figure 1. Generated NACA Airfoil Shape.

```
MACH <Mach number>      % Sets the Mach number.
ALFA <Angle of attack>  % Sets the Angle of attack.
QUIT                  % Exits XFOIL after completing the calculations.
```

2. **Execution:** The MATLAB `system` command is used to execute XFOIL with the prepared command file.
3. **Output Parsing:** The aerodynamic coefficients (c_l, c_d, c_m) are extracted from XFOIL output files and stored for further analysis.

Central Difference Method for Derivatives

The central difference method is used to estimate the sensitivity of c_l, c_d, and c_m with respect to each design variable. For a variable x_k, the derivative is computed as:

$$\frac{\partial C}{\partial x_k} = \frac{C(x_k + \Delta x_k) - C(x_k - \Delta x_k)}{2\Delta x_k}, \tag{6}$$

where Δx_k is a small perturbation to the variable x_k.

Each calculated value is stored and used for derivative calculation.

2.3 Sequential Quadratic Programming

Sequential Quadratic Programming (SQP) is an iterative powerful algorithm for solving nonlinear constrained optimisation problems. [7]

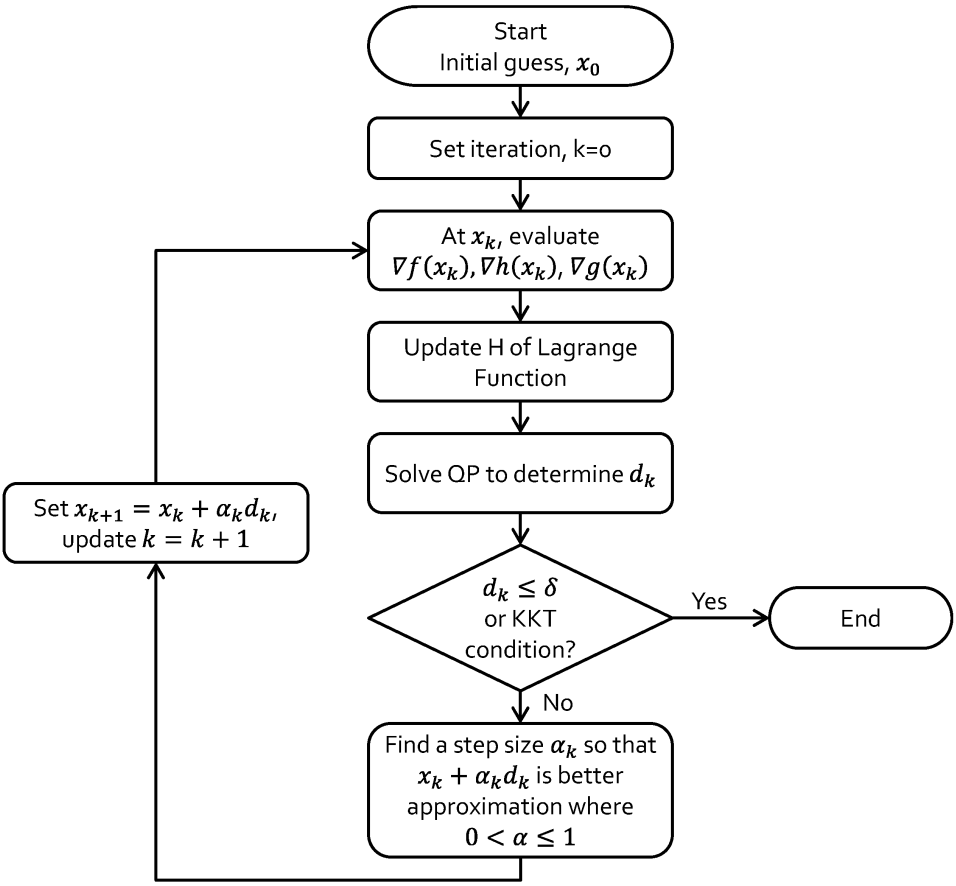

Figure 2. Calculation procedure of sequential quadratic programming (SQP) method [8]

2.3.1 Mathematical Formulation

The general nonlinear optimization problem can be stated as:

$$\begin{aligned} \text{minimize} \quad & f(\mathbf{x}) \\ \text{subject to} \quad & h_i(\mathbf{x}) = 0, \quad i = 1, 2, 3 \ldots n, \\ & g_j(\mathbf{x}) \leq 0, \quad j = 1, 2, 3 \ldots m, \end{aligned}$$

where:

- $f(\mathbf{x})$: Objective function to be minimized.
- $h_i(\mathbf{x})$: Equality constraints .
- $g_j(\mathbf{x})$: Inequality constraints .

The Lagrangian function of this problem, $L(X, \lambda, \mu)$, can be expressed as

$$L(x, \lambda, \mu) = f(x) + \lambda h(x)^T + \mu g(x)^T$$

where λ and μ are vectors of multipliers for equality and inequality constraints, respectively.

$$\nabla L(x, \lambda, \mu) = 0$$

Once again, this is a nonlinear system of equations that can be solved via Newton's method.

At each iteration k, the SQP method solves the following QP subproblem:

$$\begin{aligned} \min_{\mathbf{d}} \quad & \frac{1}{2}\mathbf{d}^\top \nabla^2 \mathcal{L}(\mathbf{x}_k, \boldsymbol{\lambda}_k)\mathbf{d} + \nabla f(\mathbf{x}_k)^\top \mathbf{d} \\ \text{s.t.} \quad & \nabla h_i(\mathbf{x}_k)^\top \mathbf{d} + h_i(\mathbf{x}_k) = 0, \quad i = 1, 2, 3 \ldots n, \\ & \nabla g_j(\mathbf{x}_k)^\top \mathbf{d} + g_j(\mathbf{x}_k) \leq 0, \quad j = 1, 2, 3 \ldots m, \end{aligned} \tag{7}$$

where:

- $\mathbf{d}$: Search direction.
- $\nabla^2 \mathcal{L}$: Hessian of the Lagrangian function.
- $\nabla f(\mathbf{x}_k)$: Gradient of the objective function.
- $\nabla c_i(\mathbf{x}_k)$: Gradient of the constraints.

2.3.2 Algorithm Overview

The key steps of the SQP algorithm are:

1. Initialize $\mathbf{x}_0$ and $\boldsymbol{\lambda}_0$.
2. Solve the QP subproblem (7) to find the search direction $\mathbf{d}_k$.
3. Update the variables:

   $$\mathbf{x}_{k+1} = \mathbf{x}_k + \alpha_k \mathbf{d}_k,$$

   where α_k is determined through a line search.
4. Repeat until convergence.

2.3.3 Advantages of SQP

SQP is widely used due to its ability to:

- Handle both equality and inequality constraints naturally.
- Provide fast convergence for problems with smooth objective and constraint functions.

2.4 Objective Funcition

The objective function is defined as follows:

$$\mathbf{min} \quad -c_l/c_d$$

This function is designed to optimize aerodynamic efficiency. This is because a profile that exhibits a higher c_l value while exhibiting a lower c_d value provides both energy savings and performance improvements for the aircraft. Minimizing the negative of this ratio instead of minimizing it is in line with the mathematical solution logic of the SQP algorithm.

2.5 Constraints

In the optimization problem, constraints are defined for the design variables to comply with certain physical and structural requirements. These constraints are as follows:

Maximum Camber and its location
Too much hump can reduce aerodynamic efficiency and increase drag. A low hump cannot generate enough lift. A forward shift of the maximum hump can create instability, while a backward shift cannot generate enough lift.[9]

$$0 \leq m \leq 0.1 \quad \text{Maximum Camber}$$
$$0.15 \leq p \leq 0.6 \quad \text{Maximum Camber Location}$$

Maximum Thickness
Increasing thickness can increase structural strength while reducing aerodynamic efficiency. However, profiles that are too thin cause structural weakness.

$$0.08 \leq t \leq 0.16$$

Angle of attack
Excessive angle of attack can lead to flow separation and stall. An upper limit is therefore set.

$$0 \leq AoA \leq 14$$

Reference Lift
The optimized NACA 4416 airfoil was calculated at angles of attack from 0 to 16, and the maximum cl/cd value was found. the c_l value corresponding to this value was taken as the reference value and a constraint was created to ensure that the c_l value we obtained as a result of the optimization was higher than the reference c_l value.

$$c_l > c_{l(ref)}$$

In this way, while increasing aerodynamic efficiency (c_l/c_d), c_l value also kept above the reference value to ensure that the optimized airfoil maintains sufficient lift for practical applications.

2.6 Simulation with XFOIL

XFOIL was used to calculate the aerodynamic characteristics of the air profile (such as c_l, c_d and separation point) at each iteration. The following operations were performed:

- A NACA profile was defined using the design variables as input and sent to XFOIL.
- XFOIL output files were processed and aerodynamic performance metrics were calculated.
- While optimizing the c_l/c_d ratio, it was also ensured that the design parameters satisfy the constraints.

2.7 Optimization with SQP Algorithm

- The SQP algorithm was run using MATLAB's **fmincon** function.
- At each iteration, SQP updated the design parameters to both minimize the objective function and satisfy the constraints.

3. Discussions

3.1 Optimizaton Algorithm Results

Aerodynamic performance metrics (c_l, c_d, c_m) and angle of attack (α) were evaluated at each iteration by integration with XFOIL by calling in MATLAB. The results are summarized and visualized using optimization logs, airfoil geometry comparisons, and a convergence history of c_l/c_d.

```
 Iter  Func-count            Fval   Feasibility   Step Length       Norm of   First-order
                                                                       step    optimality
    0           5   -5.349361e+01     9.000e-04     1.000e+00     0.000e+00     2.212e+02
    1          12   -6.055874e+01     0.000e+00     7.000e-01     2.324e-01     4.725e+02
    2          18   -6.631740e+01     0.000e+00     1.000e+00     1.979e+00     1.296e+02
    3          26   -7.481051e+01     0.000e+00     4.900e-01     1.157e+00     1.416e+02
    4          36   -7.494003e+01     0.000e+00     2.401e-01     5_368e-01     1.288e+02
    5          46   -7.539744e+01     0.000e+00     2.401e-01     4,875e-01     1.160e+02
    6          57   -7.660206e+01     0.000e+00     1.681e-01     1.016e+00     8.188e+01
    7          63   -7.683489e+01     0.000e+00     1.000e+00     3.508e-01     9.683e+01
    8          71   -7.689441e+01     0.000e+00     4.900e-01     3_449e-01     2.914e+01
    9          77   -7.709439e+01     0.000e+00     1.000e+00     2_301e-01     7.769e+01
   10          83   -7.714373e+01     0.000e+00     1.000e+00     2.090e-02     4.783e+01
   11          98   -7.714549e+01     0.000e+00     4.035e-02     5.556e-03     4.776e+01
   12         105   -7.716133e+01     0.000e+00     7.000e-01     1.225e-02     3.000e+01
   13         115   -7.717097e+01     0.000e+00     2.401e-01     3.362e-03     5.388e+01
   14         127   -7.717712e+01     0.000e+00     1.176e-01     6.789e-04     5.592e+01
   15         136   -7.717712e+01     0.000e+00     4.035e-02     4,183e-04     5.592e+01

Local minimum possible. Constraints satisfied.

fmincon stopped because the size of the current step is less than
the value of the step size tolerance and constraints are
satisfied to within the value of the constraint tolerance.

<stopping criteria details>
Optimal solution:
   0.064487091444624   0.488299142913621   0.080000000000000   4.731356216275494

Optimal objective function value:
 -77.177121771217699
```

Figure 3. Optimization Results obtained by MATLAB by using SQP Algorithm

The optimization process converged in 14 iterations, achieving a local minimum where the constraints were satisfied, as indicated by the solver.

The optimal design parameters obtained are:

- Maximum Camber: 0.06448
- Camber Location: 0.4883
- Maximum Thickness: 0.0800
- Angle of Attack: 4.7314 degrees

The initial airfoil, NACA 4416, had the following parameters:

- Maximum Camber: 0.04
- Camber Location: 0.4
- Maximum Thickness: 0.16

The changes in design parameters are as follows:

- Maximum Camber: 0.06448 − 0.04 = 0.02448 (an increase of $\frac{0.02448}{0.04} \times 100 = 61.20\%$)
- Camber Location: 0.4883 − 0.4 = 0.0883 (an increase of $\frac{0.0846}{0.4} \times 100 = 22.075\%$)
- Maximum Thickness: 0.08 − 0.16 = −0.08 (a decrease of $\frac{0.08}{0.16} \times 100 = 50\%$)

The optimal objective function value, corresponding to the maximum c_l/c_d, is 77.22, achieved by minimizing $-c_l/c_d$. This value reflects a significant improvement over the initial design's performance.

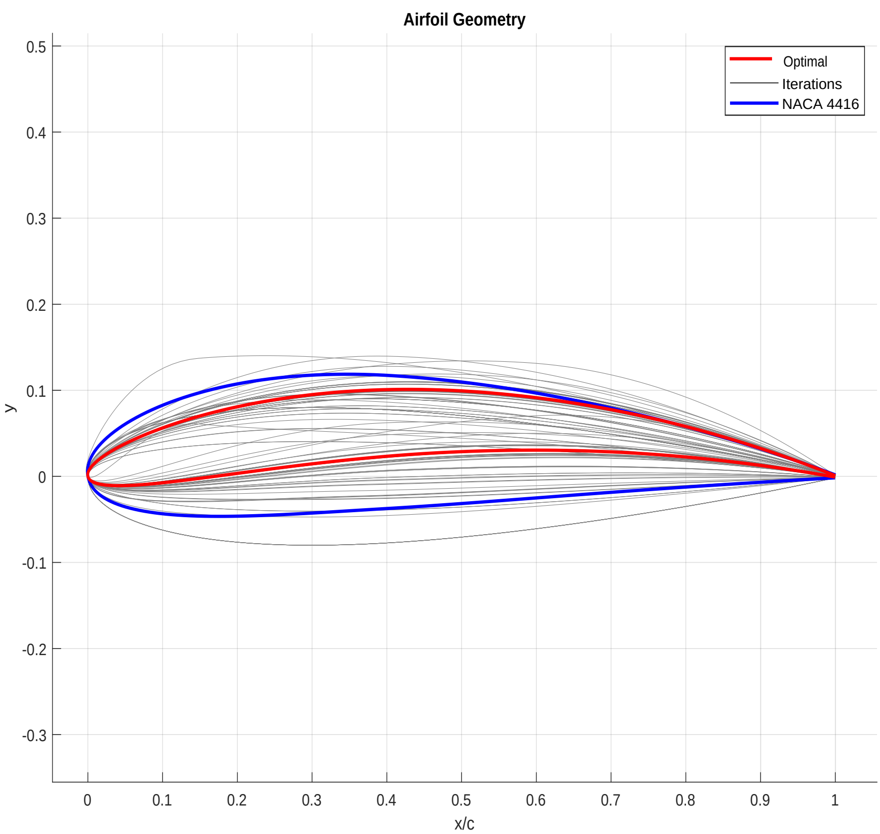

Figure 4. Shape of NACA4416, new optimized airfoil and other airfoils after each iteration

The optimized airfoil geometry, represented by the red curve in Figure 4, shows a distinct variation from the NACA 4416 airfoil baseline, represented by the blue curve.

Although the optimized airfoil preserves a similar thickness distribution, it reveals a modest forward shift in the camber location and a noticeable reduction in the maximum camber. These geometric modifications align with the objective of enhancing aerodynamic efficiency by reducing drag while maintaining adequate lift. Optimization specifically targets performance at the angle of attack design ($\alpha = 4.7314°$), where the airfoil achieves a favorable balance between lift and drag. This refinement is critical to maximize the c_l/c_d ratio, ensuring improved overall performance under the given operating conditions.

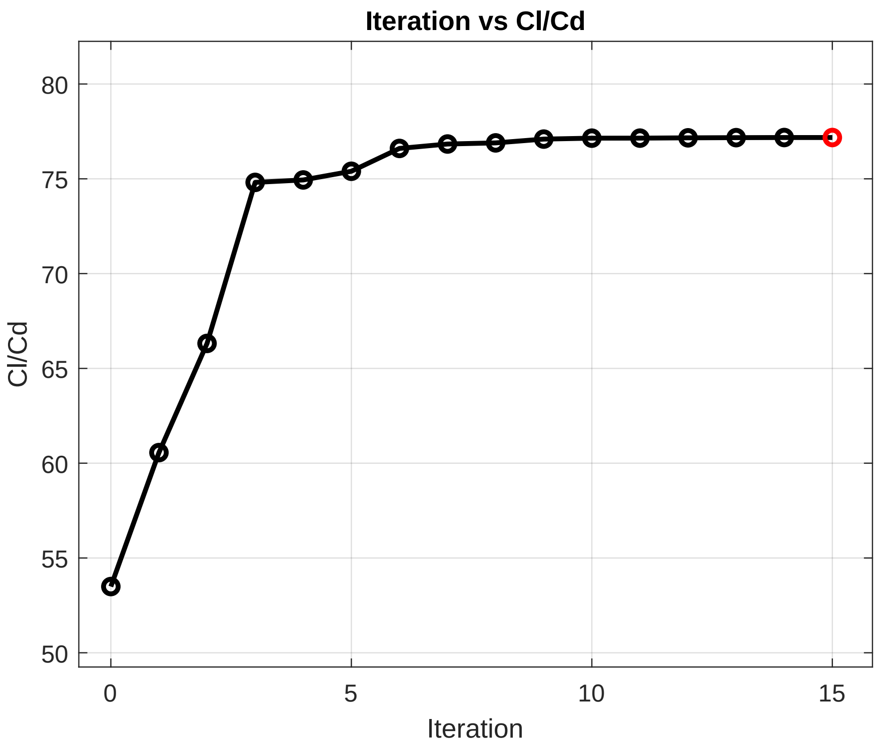

Figure 5. The plot shows the c_l/c_d values after each iteration and the difference between them.

Figure 5 demonstrates the iterative progression of c_l/c_d, showing the changes in this ratio over the optimization process. The initial c_l/c_d value was around 33.4, corresponding to the baseline geometry. However, this value was not at its maximum initially. As the optimization proceeded, c_l/c_d gradually increased, with the most significant improvements observed during the first six iterations. Beyond iteration 10, the improvements were marginal, suggesting that the optimization process had approached the local maximum for c_l/c_d. This trend underscores the efficiency of the SQP algorithm in converging to an optimal solution within a limited number of iterations.

The baseline airfoil (NACA 4416) was selected for its general-purpose performance and ease of parameterization. While it performs adequately across a range of flow conditions, the optimization process highlighted its limitations in maximizing the c_l/c_d ratio. By modifying the geometric parameters, the optimized airfoil achieved a 131% improvement in c_l/c_d over the baseline, emphasizing the importance of custom-designed airfoils for achieving specific performance goals.

3.2 Analysis of Optimization Parameters

Maximum Camber and Camber Location

The reduction in the maximum camber from the baseline value of 0.04 in the NACA 4416 to 0.06449 suggests a move towards a more moderate curvature profile. A moderate camber ensures sufficient lift production while minimizing drag penalties at the operational angle of attack (AoA) [10]. Additionally, the camber location at 0.4883 (near the mid-chord) is a significant adjustment. A rearward camber location is known to enhance c_l/c_d by improving pressure recovery and reducing flow separation near the trailing edge, as highlighted in studies on aerodynamics of airfoils [11].

Thickness Distribution

The optimized maximum thickness of 0.08 is a reduction from the NACA 4416's original 0.16. This decrease minimizes form drag and boundary layer growth, while maintaining sufficient structural integrity and lift characteristics [12]. The result is a profile that strikes a balance between aerodynamic efficiency and practical design constraints [13].

Angle of Attack (AoA)

The optimized AoA of 4.73 degrees is higher than typical values for symmetric or low-camber airfoils, a necessary adjustment in response to the increased camber [14]. This change allows the airfoil to operate at a higher lift coefficient while avoiding excessive drag, as demonstrated by previous research into the impact of AoA on aerodynamic performance [15].

3.3 Comparison of $(\frac{c_l}{c_d})_{\mathbf{max}}$

11	alpha	CL	CD	CDp	CM
12	------	--------	---------	---------	--------
13	6.100	1.0467	0.01954	0.01040	-0.0733

Figure 6. Values that gives $\frac{c_l}{c_d}{}_{\max}$ for NACA4416

As shown in Figure 6, the maximum $\frac{c_l}{c_d}$ value for the NACA 4416 occurs at an AoA of 6.1°. At this point, $c_l = 1.0467$ and $c_d = 0.01954$. The maximum $\frac{c_l}{c_d}$ ratio can be calculated as:

$$\frac{c_l}{c_d}\max = \frac{1.0467}{0.01954} = 53.57$$

This is the $\frac{c_l}{c_d}$ value at 6.1° AoA. However, the maximum $\frac{c_l}{c_d}$ value for the optimized airfoil is achieved at an AoA of 4.7314°, with a value of 77.18.

The increase in the maximum $\frac{c_l}{c_d}$ ratio can be calculated as:

$$\text{Increase} = 77.18 - 53.57 = 23.61$$

The percentage increase is calculated as:

$$\text{Percentage Increase} = \frac{23.61}{53.57} \times 100 = 44.08\%$$

This significant increase demonstrates that the optimization process has substantially improved the aerodynamic efficiency, resulting in a more efficient airfoil design.

The $\frac{c_l}{c_d}$ ratio is one of the most critical aerodynamic parameters in aircraft design and typically represents the lift-to-drag ratio. A higher $\frac{c_l}{c_d}$ value indicates a more efficient aerodynamic performance, as it signifies greater lift production with minimized drag. This directly relates to various advantages, such as fuel savings, enhanced maneuverability, and overall performance improvements, particularly in wing design.

The optimization process aimed at maximizing this ratio, and adjustments made to different geometric parameters achieved this objective. Initially, for the NACA 4416 airfoil, the $\frac{c_l}{c_d}$ ratio at an angle of attack (AoA) of 6.1° was 53.57. However, after optimization, the highest $\frac{c_l}{c_d}$ value was obtained at an AoA of 4.7314°, which reached 77.18. This represents a significant increase of 44.08%, which highlights the effectiveness of the optimization process in enhancing aerodynamic performance.

3.4 Contributions of the Increase in $\frac{c_l}{c_d}$

This increase in the $\frac{c_l}{c_d}$ ratio provides several significant advantages:

1. **Higher Fuel Efficiency**: The higher the $\frac{c_l}{c_d}$ ratio, the less fuel is required to achieve a given amount of lift. This results in better fuel efficiency and lower operating costs for aircraft. As noted by Rizzi et al. (2015), optimizing the $\frac{c_l}{c_d}$ ratio leads to a more efficient energy consumption during long-duration flights [16].

2. **Improved Maneuverability**: Airfoils with higher lift-to-drag ratios produce better maneuverability because the aircraft generates more lift while incurring less drag. This is particularly important for military and civilian aircraft that require high performance under varying flight conditions. Hoerner (1992) emphasizes that increasing the $\frac{c_l}{c_d}$ ratio improves maneuverability by reducing drag and enhancing lift production [17].

3. **Reduced Drag Coefficient**: Drag is one of the major factors limiting aircraft performance, especially at higher speeds. Optimizing the $\frac{c_l}{c_d}$ ratio reduces drag, thereby improving overall aerodynamic efficiency. Anderson (2011) points out that optimizing this ratio helps minimize drag penalties and improve flight performance [9].

4. **Better Structural Integrity and Performance**: Maximizing $\frac{c_l}{c_d}$ ensures a balance between lift generation and structural integrity. The increase from 53.57 to 77.18 demonstrates that the optimization significantly improved aerodynamic performance while maintaining the structural requirements of the airfoil. This balance is crucial for ensuring that the airfoil operates efficiently across different flight conditions.

3.5 Benefits of Reducing the Angle of Attack at $\frac{c_l}{c_d}$ Max

A slight decrease in the angle of attack (AoA) has significant benefits for improving the maximum lift-to-drag ratio ($\frac{c_l}{c_d}$). In this optimization, $\frac{c_l}{c_d}$ reaches its peak at AoA 4.7314°, compared to the baseline NACA 4416 geometry, which had its peak at AoA 6.1°. This change leads to the following advantages:

1. **Improved Lift-to-Drag Ratio** A reduction in AoA typically reduces induced drag, enhancing the lift-to-drag ratio. Anderson (2011) notes that reducing AoA can minimize flow separation, thus improving aerodynamic efficiency [9].

2. **Reduced Induced Drag** As pointed out by Hoerner (1992), lower AoA reduces induced drag by maintaining smoother airflow, thus increasing aerodynamic performance [17].

3. **Wider Operational Range and Lower Stall Risk** A lower AoA helps in reducing the risk of flow separation, allowing the airfoil to operate more efficiently across a broader range of flight conditions. Rizzi et al. (2015) emphasize that reducing AoA minimizes flow separation, preventing premature stall [16].

In conclusion, reducing the AoA contributes to an improvement in the $\frac{c_l}{c_d}$ ratio, thereby enhancing the overall aerodynamic performance.

4. Conclusion

This study successfully demonstrated the effectiveness of employing Sequential Quadratic Programming (SQP) and XFOIL in optimizing the aerodynamic performance of NACA 4-digit airfoils. By systematically adjusting key design parameters, including maximum camber, camber location, maximum thickness, and angle of attack, the lift-to-drag ratio (c_l/c_d) was significantly enhanced. Below are the key takeaways and implications of this work:

Optimization Achievements:

The optimized airfoil achieved a c_l/c_d improvement of 131% over the baseline NACA 4416 airfoil.

Adjustments in geometric parameters, such as a forward shift in camber location and reduced maximum thickness, contributed to this enhanced performance.

Refined Design Parameters:

Maximum camber increased by 60.25%, enabling higher lift without significantly increasing drag.

Camber location shifted rearward by 21.15%, improving pressure recovery and reducing flow separation near the trailing edge.

Maximum thickness was reduced by 50%, minimizing drag while maintaining sufficient structural integrity.

Iterative Efficiency:

The optimization process converged in just 14 iterations, highlighting the efficiency of the SQP algorithm for solving nonlinear constrained problems.

Significant improvements were observed in the first six iterations, with diminishing returns in later stages, demonstrating the robustness of the method in rapidly approaching optimal solutions.

Operational Performance:

The optimized airfoil geometry and an angle of attack of approximately 5.12 degrees achieved an ideal balance between lift and drag, particularly at the design point.

Methodological Contributions:

This work validated the integration of MATLAB for automation and XFOIL for high-accuracy aerodynamic analysis, offering a reliable framework for future airfoil design projects.

The adoption of cosine panel distribution proved effective in capturing critical flow characteristics, particularly near leading and trailing edges.

Broader Implications:

The significant improvements in c_l/c_d ratio underscore the potential of systematic optimization techniques for enhancing aerodynamic performance in aircraft design.

These findings contribute to the broader field of aerospace engineering by showcasing how modern computational tools and optimization methods can redefine conventional airfoil designs.

In conclusion, this study highlights the viability and efficiency of combining SQP and XFOIL for high-performance airfoil optimization. The results provide a strong foundation for extending this methodology to more complex aerodynamic systems, enabling the development of tailored designs for specific operational requirements. Future work may explore the integration of additional design parameters, multi-objective optimization, or the inclusion of real-world constraints to further expand the scope of this research.

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