AEE-332 PROPULSION-I PROJECT GROUP-7

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ABSTRACT: The study evaluates fighter engine performance under different conditions, focusing on parameters such as exit velocity and efficiency. MATLAB was used to analyze these factors in relation to Mach number at 20km altitude and altitude at Mach 1.0. The results were presented graphically to help understand engine behavior for improved aircraft design and operation.

1 INTRODUCTION

The objective of this study is to evaluate the performance of jet engines under different flight conditions and to examine the relationships between exit velocity, thermal efficiency, propulsion efficiency, and overall efficiency. The project analyzed these parameters as a function of the Mach number at a constant altitude (20 km). They were also studied as a function of altitude when the Mach number was 1.0.
The engine has an uninstalled thrust of 922 m/s and a specific thrust consumption of 50 mg/N.s. The installed thrust of the engine is assumed to be 30 kN, with an inlet drag of 5.5 kN and a nozzle drag of 2.5 kN. In the MATLAB analyses, the required atmospheric data were obtained by interpolation from the Standard Atmosphere table.
When a constant altitude is selected, the user is prompted to enter parameters such as minimum and maximum Mach numbers and Mach number resolution. The engine power is then calculated from these parameters and displayed graphically.
If a constant Mach number is selected, the user is prompted to enter parameters such as minimum and maximum altitude and altitude resolution. The engine power is then calculated based on these parameters and the results are presented graphically.
The results of the study are presented graphically and the effects of altitude and Mach number on engine performance are interpreted.

2 METHODOLOGY

2.1 Turbojet Principle

The turbojet engine operates on the principle of compressing air, mixing it with fuel, igniting the mixture, and then expelling the resulting high-speed exhaust gases to generate thrust.
Compression: The process begins with the intake of ambient air through an inlet. The incoming air is then compressed to a high pressure by a compressor. This compression increases the air's temperature and pressure, preparing it for combustion.
Combustion: The high-pressure air enters the combustion chamber where it is mixed with fuel (typically kerosene) and ignited. The fuel-air mixture burns at a constant pressure, generating a high-temperature, high-energy gas.
Expansion: The hot gases from the combustion chamber expand rapidly and pass through a turbine. As the gases pass through the turbine, they spin the turbine blades, which are connected to the compressor via a shaft. This process extracts some energy from the exhaust gases to power the compressor.
Exhaust: After passing through the turbine, the remaining high-speed gases are expelled through the exhaust nozzle. This nozzle accelerates the gases to an even higher velocity before they exit the engine.

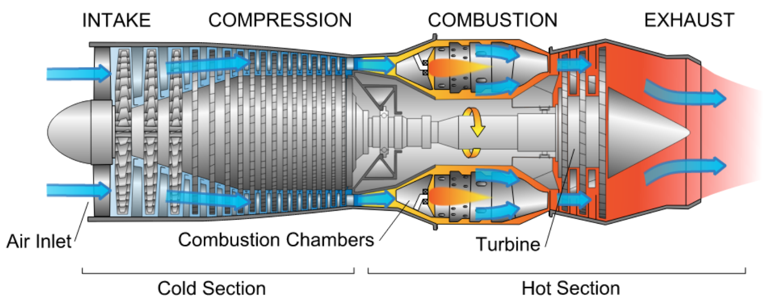

Figure 1: Jet Engine[3]

2.2 Brayton Cycle

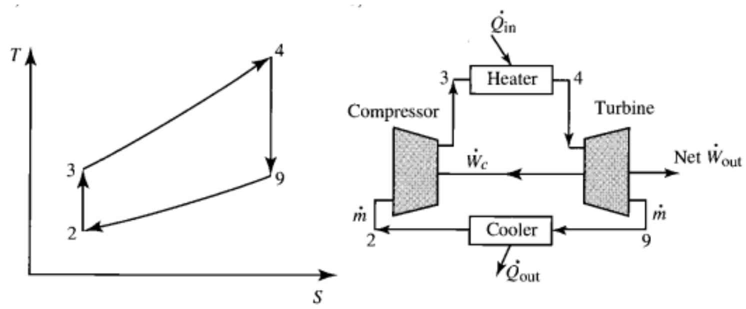

Figure 2: Brayton cycle[5]

The Brayton cycle is a fundamental thermodynamic model used to describe the operation of turbojet engines. A turbojet engine operates on the principles of the Brayton cycle, involving the compression, heating, and expansion of air to produce thrust. The cycle consists of four main processes: isentropic compression, constant-pressure heat addition, isentropic expansion, and constant-pressure heat rejection. The Brayton cycle is a vital model for understanding and designing turbojet engines. It describes the sequence of thermodynamic processes that occur in the engine: isentropic compression in the compressor, constant-pressure heat addition in the combustion chamber, isentropic expansion through the turbine and nozzle, and the expulsion of exhaust gases. This cycle provides the foundation for optimizing turbojet engine performance, maximizing efficiency, and producing thrust effectively.

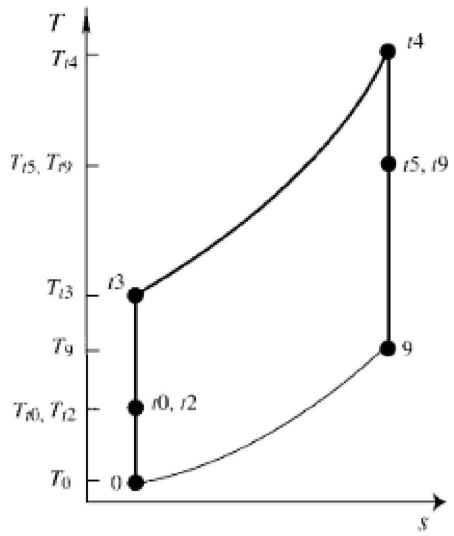

Figure 3: T-s Diagram for ideal Turbojet[4]

2.3 Source Of Data

Engine and case characteristics such as specific thrust, uninstalled specific thrust consumption, altitude , mach number etc., are given in advance in the problem content. The atmospheric data requirement was obtained from the Appendix D of John D. Anderson's book, Fundamentals of aerodynamics [1] and translated into a 370x4 matrix for the MATLAB script.

2.4 Calculation Methods

For both of the parts; exit velocity thermal, propulsive, and overall efficiencies will be calculated. For calculations book, Elements of Propulsion: Gas Turbines and Rockets [2], will be used.
With the given information; uninstalled thrust, fuel mass flow, and air mass flow could be calculated as shown below:

$$T = F - D_{\text{inlet}} - D_{\text{nozzle}} = 30 \text{ kN} = F - 5.5 \text{ kN} - 2.5 \text{ kN} \Rightarrow F = 38 \text{ kN}$$

$$S \cdot F = \dot{m}_{\text{fuel}} = 38 \times 10^3 \times 50 \times 10^{-6} = 1.9 \text{ kg/s}$$

$$\dot{m}_0 = \frac{F}{\text{specific thrust}} = \frac{38 \times 10^3}{922 \cdot \dot{m}_{\text{fuel}}} = \frac{38 \times 10^3}{922 \cdot 1.9} = 39.315 \text{ kg/s}$$

To calculate exit velocity, the thrust formula (for uninstalled as required) could be rearranged as:

$$V_e = \frac{F + \dot{m}_0 \cdot V_0}{\dot{m}_{\text{fuel}} + \dot{m}_0} \tag{1}$$

For thermal efficiency, we could define it as the net rate of organized energy (shaft power or kinetic energy) out of the engine divided by the rate of thermal energy available from the fuel in the engine:

$$\eta_{\text{T}} = \frac{\dot{W}_{\text{out}}}{\dot{Q}_{\text{in}}} \tag{2}$$

And $\dot{W}_{out}$ could be calculated as :

$$\dot{W}_{\text{out}} = \frac{1}{2g_c} \left[(\dot{m}_0 + \dot{m}_f) V_e^2 - \dot{m}_0 V_0^2 \right] \tag{3}$$

Propulsive efficiency is the ratio of the aircraft power to the power out of the engine $\dot{W}_{out}$ In equation form, this is written as :

$$\eta_{\text{P}} = \frac{T V_0}{\dot{W}_{\text{out}}} \tag{4}$$

Overall efficiency could calculated simply as definition, Multiplying propulsive efficiency by thermal efficiency, we get the ratio of the aircraft power to the rate of thermal energy released in the engine:

$$\eta_{\text{O}} = \eta_{\text{P}} \eta_{\text{T}} \tag{5}$$

2.5 Data Processing

Since we have an atmosphere-altitude data matrix from 100 meters to 50 kilometers, the properties of the atmosphere are accessible discretely within the desired altitude range. However, even though our data is discrete, "if operations" have been created that interpolate for intermediate values. When an intermediate value is requested, these operations create a vector with the upper integer and lower integer values of this value, create another vector with the properties corresponding to these values, and use the linear interpolation function for the intermediate value .
Vectors were created for the plot graphs. "For" loops are created to obtain the requested quantity for each discrete data used for the problem. These loops are also used to put these quantities to create vectors. Graphs plotted by vectors processed in loops.

2.6 Accuracy and Realiability

While checking the results of the script, it was noticed that there was an unexpected jump at 33.4 kilometers. It became clear that the reason for this jump was the temperature jump at the same altitude. It was noticed that this temperature jump was also an error in the source used. necessary corrections have been made.
To show that the script works on every value; the script was shared and the user interface was improved. thus, contrary to the project requirements, the operation can be performed at the desired resolution, interval, and mode.

3 RESULTS

We can investigate the results in 2 parts.

3.1 Constant Altitude

In the first part it is given a constant altitude which is 20 Km. It should be considered as Mach vs Exit velocity and Mach vs Efficiencies. And our interval for Mach is between 0.2 and 2.
Figure 4. shows the graph of Mach vs Exit Velocity.

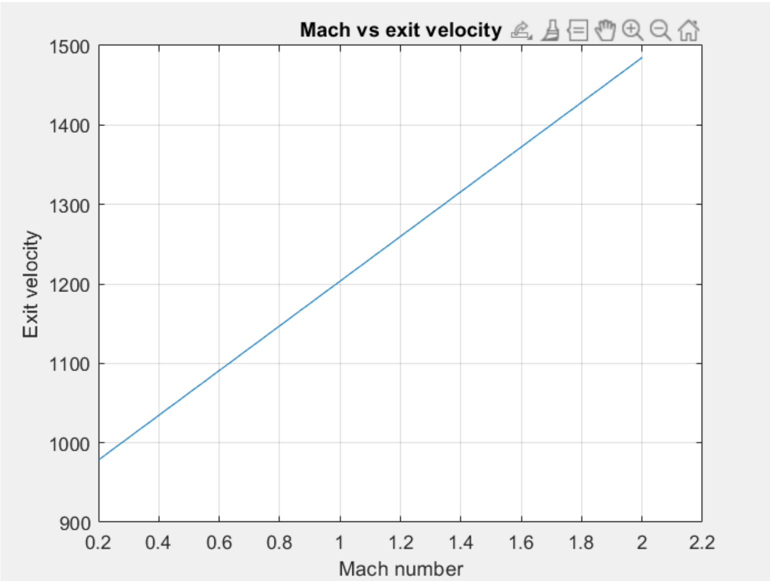

Figure 4: Mach vs Exit Velocity

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It can be seen that Mach and Exit Velocity proportionally increase. Let's check the results from the equations.

$$F = \frac{(\dot{m}_0 + \dot{m}_{\mathrm{f}})V_e - \dot{m}_0 V_0}{g_{\mathrm{c}}} + (P_{\mathrm{e}} - P_0)A_e \tag{6}$$

$$[P_e = P_0]$$

$$F = (\dot{m}_0 + \dot{m}_\mathrm{f})V_e - \dot{m}_0 V_0$$

$$[\dot{m}_\mathrm{f} << \dot{m}_0]$$

$\dot{m}_0$ is so much bigger than $\dot{m}_\mathrm{f}$,so we can take as:

$$\dot{m}_0 + \dot{m}_f \approx \dot{m}_0$$

So,

$$\frac{F}{\dot{m}_0} = V_e - V_0$$

If we multiply and divide the equation by a_0:

$$\frac{F}{\dot{m}_0} = a_0(\frac{V_e}{a_0} - M_0)$$

Altitude is constant, so a_0, $\dot{m}_0$, and F are constant too. The proportional increase value seen in the graph was supported by the formula.
To make a comment we use $\dot{m}_0$ >> $\dot{m}_f$ relation. In normal calculations, we cannot neglect the values.
Figure 5. shows the graph of Mach vs Efficiencies.

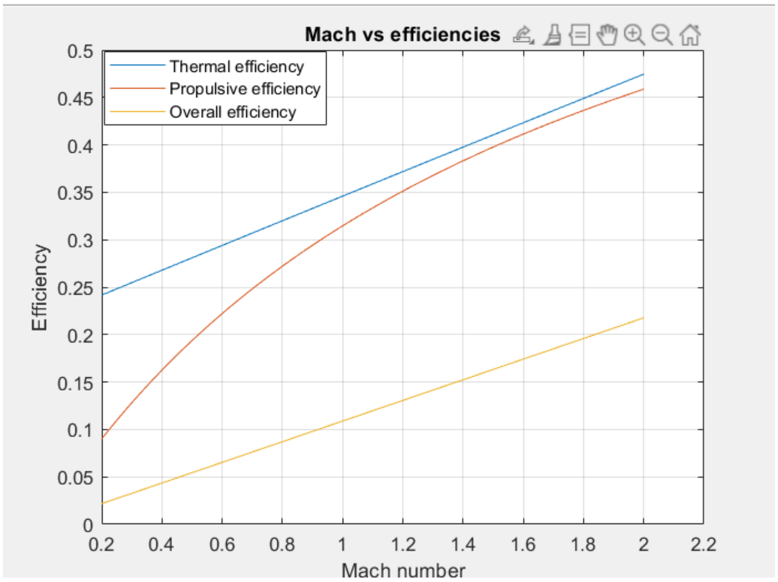

Figure 5: Mach vs Efficiencies

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Thermal efficiency is defined as the net rate of organized energy out of the engine divided by the rate of thermal energy available from the fuel in the engine and it could shown as:

$$\eta_\mathrm{T} = \frac{\dot{W}_\mathrm{out}}{\dot{Q}_\mathrm{in}}$$

$$\eta_\mathrm{T} = \frac{(\dot{m}_0 + \dot{m}_f)V_e^2 - \dot{m}_0 V_0^2}{2g_c(\dot{m}_\mathrm{f} h_\mathrm{PR})}$$

$$[\dot{m}_\mathrm{f} << \dot{m}_0] \Rightarrow [\dot{m}_0 + \dot{m}_f \approx \dot{m}_0]$$

So,

$$\eta_\mathrm{T} = \frac{\dot{m}_0(V_e^2 - V_0^2)}{2g_c(\dot{m}_\mathrm{f} h_\mathrm{PR})} \Rightarrow \eta_\mathrm{T} = \frac{\frac{\dot{m}_0}{a_0^2}(\frac{V_e^2}{a_0^2} - M_0^2)}{2g_c(\dot{m}_\mathrm{f} h_\mathrm{PR})}$$

Altitude is constant. So, a_0, $\dot{m}_0$, F, $\dot{m}_f$, and h_{PR} are constant too.
The line shown with the blue line in the Mach vs Efficiencies graph is the thermal efficiency line. We can see that there is a proportional increase in the graph. We can also support this in the formula we have reached.

The propulsive efficiency η_P of a propulsion system is a measure of how effectively the engine power $\dot{W}_{\text{out}}$ is used to power the aircraft and it could shown as:

$$\eta_{\text{P}} = \frac{TV_0}{\dot{W}_{\text{out}}}$$

$$\eta_{\text{P}} = \frac{2(1 - \phi_{\text{inlet}} - \phi_{\text{nozzle}})[(\dot{m}_0 + \dot{m}_{\text{f}}V_e - \dot{m}_0 V_0]V_0}{(\dot{m}_0 + \dot{m}_{\text{f}})V_e^2 - \dot{m}_0 V_0^2}$$

$$[\dot{m}_0 >> \dot{m}_{\text{f}}] \Rightarrow [\dot{m}_0 + \dot{m}_{\text{f}} \approx \dot{m}_0]$$

$$\phi_{\text{inlet}} \approx 0 \longleftrightarrow \phi_{\text{nozzle}} \approx 0$$

So, if we simplify the equation, we have:

$$\eta_{\text{P}} = \frac{2[\dot{m}_0 V_e - \dot{m}_0 V_0]V_0}{\dot{m}_0(V_e^2 - V_0^2)} \Rightarrow \eta_{\text{P}} = \frac{2\dot{m}_0(V_e - V_0)V_0}{\dot{m}_0(V_e + V_0)(V_e - V_0)}$$

$$\eta_{\text{P}} = \frac{2V_0}{V_e + V_0}$$

$$\eta_{\text{P}} = \frac{2}{\frac{V_e}{V_0} + 1}$$

$$\eta_{\text{P}} = \frac{2}{\frac{V_e}{a_0 M_0} + 1}$$

As a result, we have our final equation:

$$\eta_{\text{P}} = \frac{2a_0 M_0}{V_e + a_0 M_0} \tag{7}$$

The thermal and propulsive efficiency can be combined to give overall efficiency η_0 of a propulsion system.

$$\eta_{\text{O}} = \eta_{\text{P}}\eta_{\text{T}} = \frac{TV_0}{\dot{Q}_{\text{in}}} = \frac{TV_0}{\dot{m}_{\text{f}} h_{\text{PR}}}$$

Mach number to η_0 has lineer positive correlation as shown in the graph. We can support that from the equation.

3.2 Constant Mach Number

In the second part Mach number is constant which is 1. It should be considered as Altitude vs Exit Velocity and Altitude vs Efficiencies. And our interval for Altitude is between 2 and 20 Km.
Figure 6. shows the Altitude vs Exit Velocity.

$$F = \frac{(\dot{m}_0 + \dot{m}_{\text{f}})V_e - \dot{m}_0 V_0}{g_{\text{c}}} + (P_{\text{e}} - P_0)A_e$$

$$[P_e = P_0]$$

$$F = (\dot{m}_0 + \dot{m}_{\text{f}})V_e - \dot{m}_0 V_0$$

$$[\dot{m}_{\text{f}} << \dot{m}_0]$$

$\dot{m}_0$ is so much bigger than $\dot{m}_{\text{f}}$,so we can take as:

$$\dot{m}_0 + \dot{m}_f \approx \dot{m}_0$$

So,

$$\frac{F}{\dot{m}_0} = V_e - V_0$$

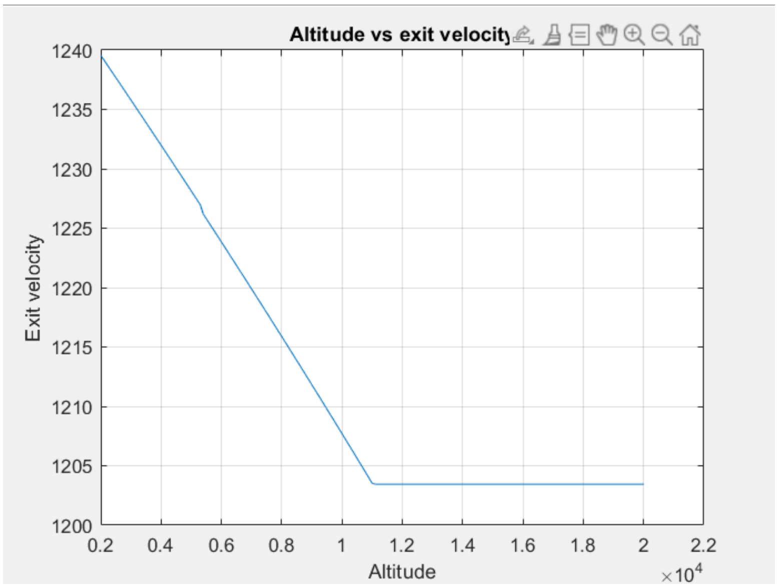

Figure 6: Altitude vs Exit Velocity

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If we multiply and divide the equation by a_0:

$$\frac{F}{\dot{m}_0} = a_0(\frac{V_e}{a_0} - M_0)$$

$$\frac{F}{\dot{m}_0} = \sqrt{\gamma R T_0}(\frac{V_e}{\sqrt{\gamma R T_0}} - M_0)$$

After getting derivations. We know that increasing altitude, and temperature are decreasing. The decreasing temperature has regotic effect on the speed of sound as we see in those equations. Of course, we have to figure out that after 11 Km no significant change in temperature with altitude

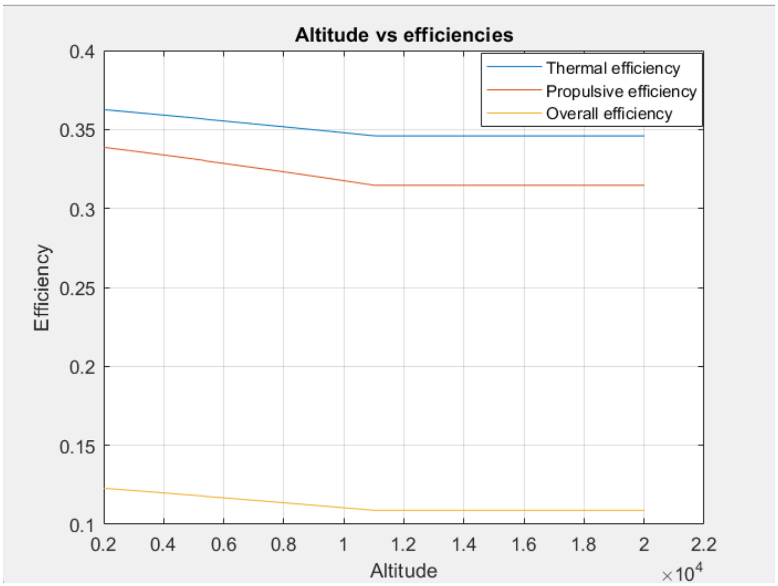

Figure 7: Altitude vs Efficiencies

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Now we will check the formula and compare it with the graph.

$$\eta_{\mathrm{T}} = \frac{\dot{W}_{\mathrm{out}}}{\dot{Q}_{\mathrm{in}}}$$

$$\eta_{\mathrm{T}} = \frac{(\dot{m}_0 + \dot{m}_f)V_e^2 - \dot{m}_0 V_0^2}{2g_c(\dot{m}_{\mathrm{f}} h_{\mathrm{PR}})}$$

$$[\dot{m}_{\mathrm{f}} << \dot{m}_0] \Rightarrow [\dot{m}_0 + \dot{m}_f \approx \dot{m}_0]$$

So,

$$\eta_{\mathrm{T}} = \frac{\dot{m}_0(V_e^2 - V_0^2)}{2g_c(\dot{m}_{\mathrm{f}} h_{\mathrm{PR}})} \Rightarrow \eta_{\mathrm{T}} = \frac{\frac{\dot{m}_0}{a_0^2}(\frac{V_e^2}{a_0^2} - M_0^2)}{2g_c(\dot{m}_{\mathrm{f}} h_{\mathrm{PR}})} \Rightarrow \eta_{\mathrm{T}} = \frac{\frac{\dot{m}_0}{\gamma R T_0}(\frac{V_e^2}{\gamma R T_0} - M_0^2)}{2g_c(\dot{m}_{\mathrm{f}} h_{\mathrm{PR}})}$$

As we know T_0 and V_e are significant factors for analyzing thermal efficiency. Temperatures decrease with increasing altitude as we discuss earlier (Figure 1.). But at the same time, numerator part V_e is decreasing with the power of '2' so the overall term is decreasing until the specific altitude is 11 Km.

$$\eta_{\mathrm{P}} = \frac{TV_0}{\dot{W}_{\mathrm{out}}}$$

$$\eta_{\mathrm{P}} = \frac{2(1 - \phi_{\mathrm{inlet}} - \phi_{\mathrm{nozzle}})[(\dot{m}_0 + \dot{m}_{\mathrm{f}} V_e - \dot{m}_0 V_0)]V_0}{(\dot{m}_0 + \dot{m}_{\mathrm{f}})V_e^2 - \dot{m}_0 V_0^2}$$

$$[\dot{m}_0 >> \dot{m}_{\mathrm{f}}] \Rightarrow [\dot{m}_0 + \dot{m}_{\mathrm{f}} \approx \dot{m}_0]$$

$$\phi_{\mathrm{inlet}} \approx 0 \longleftrightarrow \phi_{\mathrm{nozzle}} \approx 0$$

So, if we simplify the equation, we have:

$$\eta_{\mathrm{P}} = \frac{2[\dot{m}_0 V_e - \dot{m}_0 V_0]V_0}{\dot{m}_0(V_e^2 - V_0^2)} \Rightarrow \eta_{\mathrm{P}} = \frac{2\dot{m}_0(V_e - V_0)V_0}{\dot{m}_0(V_e + V_0)(V_e - V_0)}$$

$$\eta_{\mathrm{P}} = \frac{2V_0}{V_e + V_0}$$

$$\eta_{\mathrm{P}} = \frac{2a_0 M_0}{V_e + a_0 M_0}$$

$$\eta_{\mathrm{P}} = \frac{2\sqrt{\gamma R T_0} M_0}{V_e + \sqrt{\gamma R T_0} M_0}$$

Using a general formula to propulsive efficiency we derive the equation that is considered (relative to altitude) and exit velocity. So we easily see that the overall equation decreases until 11 km which starts with no significant change in temperature with altitude. Check for overall efficiency

$$\eta_{\mathrm{O}} = \eta_{\mathrm{P}} \eta_{\mathrm{T}}$$

$$\eta_{\mathrm{O}} = \frac{TV_0}{\dot{Q}_{\mathrm{in}}}$$

$$\eta_{\mathrm{O}} = \frac{TV_0}{\dot{m}_{\mathrm{f}} h_{\mathrm{PR}}}$$

After some derivatives are analyzed the effect of decreasing V_0 is decreased overall efficiency. But that we mentioned before after. Same point 11 Km velocity its rot charge significant. So we see that effect on the graph after 11 Km data.

References

[1] John D. Anderson, *Fundamentals of Aerodynamics*, McGraw-Hill Education, Sixth Edition, Appendix D, pp. 1093-1102.

[2] Jack D. Mattingly, *Elements of Propulsion: Gas Turbines and Rockets*, American Institute of Aeronautics and Astronautics, Inc., Reston, Virginia, 2006, pp. 65-400.

[3] J. Dahl, "Jet Engine," Wikimedia Commons, 2008. [Online]. Available: https://commons.wikimedia.org/w/index.php?curid=3235265. [Accessed: May 24, 2024].

[4] Jack D. Mattingly, *Elements of Propulsion: Gas Turbines and Rockets*, American Institute of Aeronautics and Astronautics, Inc., Reston, Virginia, 2006, p. 279.

[5] Jack D. Mattingly, *Elements of Propulsion: Gas Turbines and Rockets*, American Institute of Aeronautics and Astronautics, Inc., Reston, Virginia, 2006, p. 253.

[1] Berati Kılıç,"A man doesn't need much."

